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Theory and Methodology

The inclusion of 'spinning reserves' in investment and simulation models for electricity generation

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Abstract: The demand for electricity varies substantially from hour to hour, from day to day, and from season to season. Because electricity is a highly perishable commodity which can be stored only at great expense, and because the penalty for failing to meet demand is severe, the fluctuations of demand over time are crucial in determining an optimal investment strategy for generating equipment. Traditionally, demand is represented by a load–duration curve which neglects the time of day when peaks and troughs occur. By incorporating a load–distribution curve into the model in a computationally tractable manner, this paper demonstrates that the results of the traditional models may be sensitive to changes in the assumptions implied by the load–duration curve.

Keywords: Mixed-integer linear programming, representation of fluctuating demand, cost minimisation.

1. Introduction

The demand for electricity varies substantially from hour to hour, from day to day, and from season to season. Because electricity must be supplied on demand and can be stored only at great expense, and because the penalty for failing to meet demand is severe, the fluctuations of demand over time are crucial in determining an optimal investment strategy for generating equipment. The purpose of this paper is to analyze the joint dispatching and capacity expansion problem. The proposed model is a mixed-integer linear programming model in the tradition of Massé and Gibrat (1957), except for its treatment of demand, which we characterize by megawatts (MW) by time of day.

The gains in realism which result from representing demand for electricity by time of day are obtained at the cost of added complexity. Computational efficiency may force the planner considering the use of this model to make the trade-off between more realism in one area for less in another. The proposed model is, therefore, not intended as a replacement for proven traditional

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capacity expansion models. Rather, it serves to complement these existing tools in situations when the time of day when a given level of demand occurs, warm-up lead times, and warm-up and cool-down costs are likely to be significant factors in determining an optimal capacity expansion strategy.

2. The dispatching and the capacity expansion problem

The short run operating costs and the system's reliability depend on the characteristics of the available generating equipment. The short run objective consists in optimizing the utilization of an existing array of generators in order to meet short term demands at minimum cost. This will be called the dispatching problem. The long run objective is to optimize the expansion of electricity generating capacity over a planning horizon of several years. This will be called the capacity expansion problem. The two problems share an obvious relationship—the short run operating cost of the dispatching problem depends upon the investment decisions of the capacity expansion problem. An optimal solution to the latter requires, therefore, the simultaneous consideration of the long run and the short run problem.

The departmental subdivisions within electric utilities reflect the complexity of integrated decision making by establishing a planning department at corporate headquarters with a planning horizon measured in years, and a staff of dispatchers at the plant and systems control making decisions involving units of time measured in minutes. In considering the capacity expansion problem, it is impossible to do minute by minute planning. Therefore, process analysis models of electricity generation have simplified the problem by assuming away any cost significance of the time when peaks and troughs of demand occur. This allows whole seasons to be condensed and characterized by a single 'load-duration curve'. This curve aggregates demand levels and plots them against the duration of time for which that

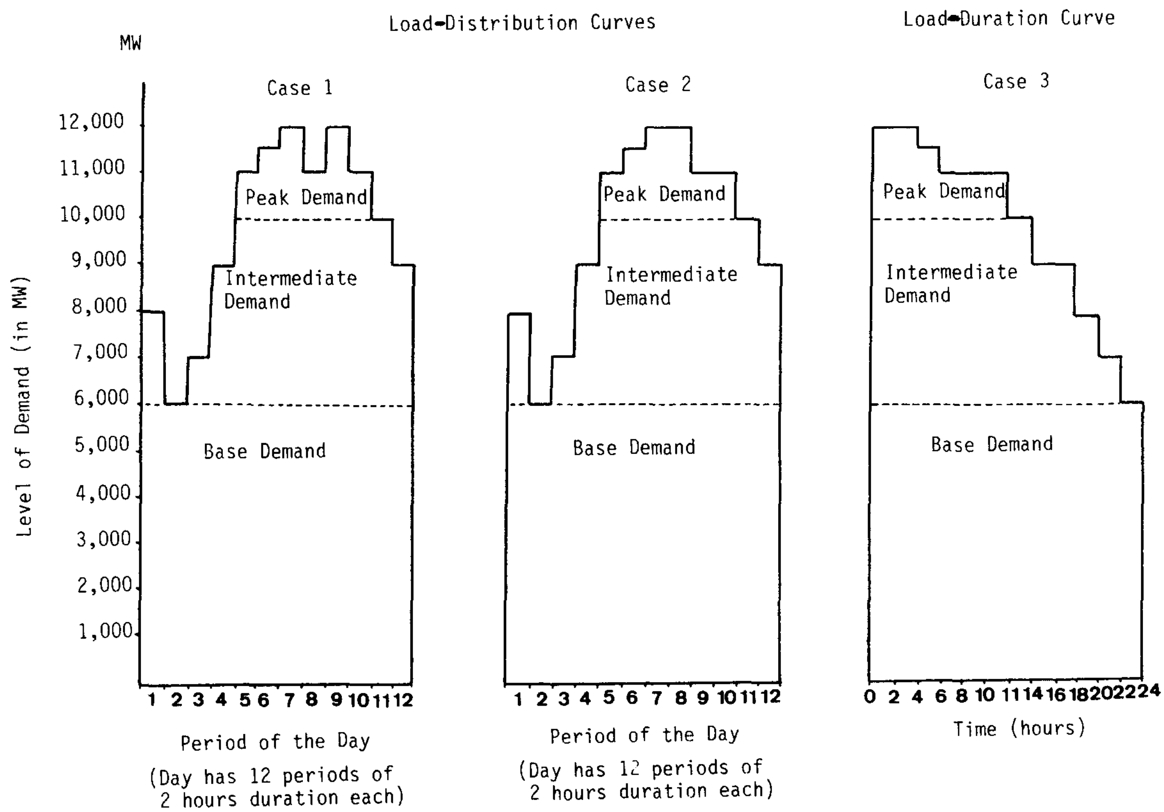

Figure 1. Representing demand: load-distribution curves vs. the load-duration curve

level or higher is demanded throughout the season. Thus, a 'load-distribution curve' (load profile) which plots time of day agains demand may be converted into a load-duration curve. This correspondence, however, is not one-to-one. Different load distribution curves may generate the same load duration curve. Figure 1 illustrates the relationship between the load-distribution and the load-duration curve. In this example, the same load-duration curve (Case 3) is derived from either the double-peaked or single-peaked load-distribution curve (Case 1 and Case 2, respectively).

Even though the dispatcher must use the load profile for the choice of generator utilization, the planner usually uses the load-duration curve when planning the capacity expansion program. In spite of this dichotomy, process analysis models of the investment problem of an electric utility have been successful and have yielded many insights. One of the first contributions to the literature, and perhaps the most influential one, was by Massé and Gibrat (1957). The works of Gately (1970), Cavalieri et al. (1971), Deonigi and Engel (1973), Manne (1974), McNamara (1976), and Sherali et al. (1984) are representative for the many extensions that have been added to Massé and Gibrat's problem formulation.

Models utilizing load-duration curves contain the implicit assumption that the efficient utilization of various generator types is determined primarily by capital and fuel costs. The costs of warming up and cooling down a plant and such intertemporal constraints as warm-up lead time do not enter the decision to invest. This assumption is a good approximation of reality in many applications. So long as those plants which are most expensive to warm up in preparation for generating also have the longest warm-up lead times and are the plants with the lowest marginal fuel cost, such simplifying assumptions will not greatly affect the optimal investment solution. These plants, therefore, are utilized for generation of the base load. Their total warm-up and cooldown costs are low because they are turned off only infrequently for scheduled maintenance. There are instances, however, when the assumptions implied by the load duration curve are not appropriate, and when the time of day when the peaks and troughs of demand occur are important. Liquid fuel combined cycle generators (type $j = 10$ in Table A.1), for example, are dominated by liquid fuel steam generators and will, therefore, never appear as a solution in a problem formulation that uses load-duration curves. The short warm-up lead time and low spinning costs of the former, however, make it a viable technology, when the dynamics of the load-distribution curve are considered. Such dynamics are also important if one wants to evaluate the potential benefits which may be obtained from new technologies (e.g., solar power—see EPRI, 1982), for which the output varies systematically over the course of a day. The distribution of demand over time can also be important for the assessment of various forms of storage of electricity where off-peak storage may reduce the warm-up costs of plants that would otherwise be shut down. In a multi-period model this may also apply to plants originally designed for base load generation but which are assigned to cyclic duty as they age and are gradually replaced.

3. The model

3.1. Fluctuations of demand by time of day

The difference between this and other models of the same genre is the way in which demand is incorporated. A season is characterized not by a load-duration curve, but by a single load profile curve that repeats itself many times. In other words, it is assumed that one load-distribution cycle can be found that repeats itself throughout each season. Demand will be characterized by one such demand cycle that represents a typical day or week of each season. The resulting sacrifice in accuracy is no greater than if one were to use a load-duration curve. In both cases, accuracy can be increased only at the expense of added complexity and a resulting loss in computational efficiency. In comparison to models utilizing a load-duration curve, the approach chosen here leads to greater complexity in the demand constraints, but the load-distribution curve contains more information regarding the dynamics of demand.

It is assumed that the cost of shifting utilization from one seasonal load pattern to another is small compared to the total operating cost during the season, so that ignoring the change-over cost has little effect on the optimal solution. This implies that, to the dispatcher, a season is infinitely long.

This assumption is common to all investment models in the tradition of Massé and Gibrat (1957). The method, used to minimize cost per cycle over an infinite number of identical cycles of (demand) constraints, is referred to here as cyclical programming. This technique allows the dynamics of demand to be characterized while keeping the number of variables as small as possible.

The load–distribution cycle must be divided into time intervals so that a stepwise approximation of the curve is obtained (see Figure 1). This curve defines the level of demand for electrical power in megawatts (MW) for each time interval. The number of time intervals necessary for an acceptable stepwise approximation of the load–distribution curve may vary from season to season. We shall index the number of time periods needed by $h = 1, 2, \dots, H(s)$, where $s = 1, 2, \dots, S$ denotes a particular season. It is assumed that the same cycle of demand recurs throughout the season. The number of seasons per year (S) and the number of time periods per cycle ($H(s)$) must be tailored to fit the demand characteristics of each application in order to minimize the number of variables and, hence, to optimize the efficiency of the solution algorithm. There are NC(s) such cycles (hereafter referred to as 'days') in season s. The level of demand during the h-th time interval ('hour') of the cycle for season s of fiscal year y is denoted $D[h, s, y]$, and is measured in megawatts. The time duration of $h = 1, 2, \dots, H(s)$ and $s = 1, 2, \dots, S$ need not be uniform, allowing further tailoring of the model. Depending on the shape of the season's load–distribution curve, choosing nonuniform time durations h may help reduce the size of the problem without sacrificing realism.

3.2. Generating equipment and production activities

Individual generating units shall be identified by plant type which is indexed $j = 1, 2, \dots, J$. The maximum capacity of plant type j during time period h of season s is denoted $K[j, h, s]$. These capacities may depend on the time of day and on the season. Lower bounds on generation, KL $[j, h, s]$ are also established. The number of plants of type j in the system during year y is denoted $R[j, y]$. Maintaining a stock of a particular plant type is subject to a maintenance cost of CR$[j, y]$ per plant.

In order to be made ready for generating electricity, a plant must be turned on and warmed up. These two activities are represented by the variable ON$[j, h, s, y]$. The time which elapses between the time when a plant is turned on and the time when it is ready to produce electricity can be substantial. It varies between several minutes for peak load diesel plants to several hours for thermal units. The warm-up lead time is an important technical parameter. It is represented by the symbol LT(j, h, s). Because the duration of the time periods (h) can vary from season to season, these lead times have to be tailored for each time period and season.

ON$[j, h, s, y]$ is the number of plants of type j which are turned on during interval h of the cycle for season s and year y. When the warm-up process is completed and the plant is ready for generating, it is said to be in its 'spinning' state. For example, a thermal plant with a hot boiler which requires only the start-up of a turbine to begin generating is in this stage. The number of plants of type j spinning during a particular time interval is denoted SPIN$[j, h, s, y]$.

Only a plant that is currently in a spinning state can generate electricity at a rate bounded by KL$[j, h, s]$ and $K[j, h, s]$. The aggregate rate of generation for all plants of type j is given by $G[j, h, s, y]$. The unit cost of generating electricity is CG$[j, h, s, y]$. The cost of warming up a plant of type j is denoted CON$[j, h, s, y]$. The unit cost of keeping a plant in its spinning state is assumed to be equal to the cost of generating electricity at the lower bound KL$[j, h, s]$.

3.3. Utilization constraints and capacity expansion

The dynamics of the utilization of the generating capacity are controlled by the warm-up schedule. A plant can spin during period h of the day if and only if it was either spinning during the previous period, $h - 1$, or if it has been warming up for the requisite lead time and reaches its spinning state during period h. This constraint is expressed by equations (1a) and (1b). We adopt the convention that $h - 1 = H(s)$ for $h = 1$, that is, the period preceding the first period of the day is the last period of the previous day. In defining the typical day, care must be taken that $H(s) + h \geqslant \text{LT}(j, h, s)$. The length of the typical day has to be sufficient to allow for the warming up of the plant with the longest lead time. Thus, for previous day warm-up,

$$\text{SPIN}[j, h, s, y] \leqslant \text{SPIN}[j, h-1, s, y] + \text{ON}[j, H(s)+h-\text{LT}(j, h, s), s, y]$$
$$\text{for } h \leqslant \text{LT}(j, h, s) \text{ and for all } j, s \text{ and } y, \tag{1a}$$

and, for all warm-up during the same day,

$$\text{SPIN}[j, h, s, y] \leqslant \text{SPIN}[j, h-1, s, y] + \text{ON}[j, h-\text{LT}(j, h, s), s, y]$$
$$\text{for } h > \text{LT}(j, h, s) \text{ and for all } j, s, \text{ and } y. \tag{1b}$$

The spinning variables SPIN[j, h, s, y] are restricted to nonnegative integer values. It is also clear that, if a plant is turned on and readied for generating, it will be in its spinning state LT(j, h, s) periods later. Again, for previous day warm-up,

$$\text{SPIN}[j, h, s, y] \geqslant \text{ON}[j, H(s)+h-\text{LT}(j, h, s), s, y]$$
$$\text{and integer-valued for } h \leqslant \text{LT}(j, h, s) \text{ and for all } j, s, \text{ and } y, \tag{2a}$$

and, for all warm-up during the same day,

$$\text{SPIN}[j, h, s, y] \geqslant \text{ON}[j, h-\text{LT}(j, h, s), s, y]$$
$$\text{and integer-valued for } h > \text{LT}(j, h, s) \text{ and for all } j, s, \text{ and } y. \tag{2b}$$

The SPIN variables are the only ones constrained to integer values. However, since turning on only a fraction of a plant yields no increase in the number of plants spinning and ready to generate electricity, a cost minimizing strategy will only allow integer values for the number of plants turned on (ON[j, h, s, y]) as well.

Two basic plant utilization strategies are available to the dispatcher—the spinning of a plant over a prolonged period of time at varying rates of generation, or variable utilization of a plant by turning it on and off. A trade-off exists between the warm-up and the spinning costs. The warm-up lead time is another important consideration in deciding on a plant utilization strategy. Base load plants that operate around the clock never incur warm-up costs in this formulation. So long as the initial warm-up cost at the beginning of the season is small compared to the season's operating costs, the elimination of the cost of switching from one season to another should not significantly affect the results. The assumption of a fixed operating pattern for each day of each season implies another simplifying property: a plant that is warmed up during a day is cooled down during the same day. Thus, the cost coefficient CON[j, h, s, y] is, without loss of generality, the sum of warm-up and cool-down costs.

The number of plants which are turned on to be readied for generation plus the number of plants spinning at that time cannot exceed the total number of plants that are maintained in the system, $R[j, y]$. These are intermediate variables, determined by both the initial stock of generation equipment and investment decisions of the future and the past (see below). The availability constraint is represented by the following inequality:

$$\text{ON}[j, h, s, y] + \text{SPIN}[j, h, s, y] \leqslant R[j, y] \quad \text{for all } j, h, s, \text{ and } y. \tag{3}$$

The number of plants available to the dispatcher in year y is equal to the number of plants available in year $y-1$, plus the number of new plants added to the system, minus the number of old plants which have reached the end of their useful economic life. The useful economic life of a plant, UL(j), is measured from the time a plant becomes available. The time that passes between the ordering of a new plant and the time when it first becomes available can be very long. This lead time is denoted LI(j). Plants of type j which become available for the first time in year y must, therefore, have been ordered in the year y − LI(j).

After UL(j) years of service, the plant must be eliminated from the system. A more general approach would allow for increased maintenance cost as a plant gets older. The determination of the useful economic life of a plant would then be endogenous to the model. This generalization, however, can be obtained only at the cost of introducing additional variables. The present approach, therefore, assumes that plants operate with the same efficiency for the duration of their useful economic life. The number of plants of type j that are in the system in year y is, therefore, given by the following equality:

$$R[j, y] = R[j, y-1] + I[j, y - \mathrm{LI}(j)] - I[j, y - \mathrm{UL}(j) - \mathrm{LI}(j)] \quad \text{for all } j \text{ and } y. \tag{4}$$

The number of plants of type j that are ordered in year y is denoted $I[j, y]$. So long as the present value of the cost of procuring new plants increases with y and the initial stock of generators, $R[j, 0]$, $j = 1, 2, \ldots, J$, is integer-valued, both new plant orders and available generators will be integer-valued at the optimum. Orders for fractions of plants will not increase spinning reserves and, therefore, are postponed in order to reduce maintenance and acquisition costs.

3.4. Demand and generating constraints

Generating electricity at a rate sufficient to keep up with demand is the first responsibility which constrains operating policy. The sum of all electricity generated by different plant types must therefore be sufficient to meet demand:

$$\sum_{j=1}^{J} G[j, h, s, y] \geqslant D[h, s, y] \quad \text{for all } h, s, \text{ and } y. \tag{5}$$

Generating capacity, of course, has its upper bounds. One cannot obtain electricity from a plant that is not warmed up and spinning. Even then, only the plant's rated capacity is available:

$$G[j, h, s, y] \leqslant \mathrm{SPIN}[j, h, s, y] K[j, h, s] \quad \text{for all } j, h, s, \text{ and } y. \tag{6}$$

Were it required that each plant generate either nothing or an amount above the output level, $\mathrm{KL}[j, h, s]$, another integer variable would have to be introduced. This additional complexity is avoided by assuming that a spinning plant must generate electricity at a level $\mathrm{KL}[j, h, s]$ or above:

$$G[j, h, s, y] \geqslant \mathrm{SPIN}[j, h, s, y] \mathrm{KL}[j, h, s] \quad \text{for all } j, h, s, \text{ and } y. \tag{7}$$

The electric utility must try to always meet demand. The planner who is responsible for the capacity expansion and replacement schedule faces a difficult problem. Considerable uncertainty exists about the level and distribution of future demand, equipment reliability, the length of lead times for new equipment, and about possible changes in regulation which may affect plant efficiency. The probability of a failure to meet demand cannot be reduced to zero. At best, it is possible to control this probability and keep it small. A number of reliability criteria have been developed for this purpose (Munasinghe, 1979). The simplest one is the establishment of a reserve margin above the expected peak demand. More sophisticated approaches use probabilistic concepts. One such concept is the loss-of-load expectation, that is, the average number of days during which daily peak demand is expected to exceed generating capacity. On day in ten years is usually regarded as an acceptable level for such expectation. Elaborations of reliability criteria consider the severity of the failure to meet demand in terms of the expected duration of an outage and the expected amount of energy not supplied because of outages (Munasinghe, 1979).

A sophisticated probabilistic approach to the system's reliability greatly adds to the complexity of the model (see, for example, Manne, 1974; Noonan and Giglio, 1977; Munasinghe, 1979). A realistic treatment would have to deal not only with the different sources of uncertainty which could lead to the system's failure to meet demand, but also with the permissible responses of the system when facing an imminent outage. Possible responses include load shedding and voltage and frequency reductions. This would add greatly to the number of variables and introduce nonlinearities into this or any other capacity expansion model. In order to keep the model computationally tractable, reserve margins are used to deal with uncertainty. Given the obvious shortcomings of this approach, it is desirable that the reliability of the recommended plant mix is analyzed separately using probabilistic concepts.

In order to assure the reliable operation of the system, it is not sufficient to maintain generating capacity in excess of expected demand. Some percentage excess capacity, m, must be ready to take the place of another generating unit immediately after the forced outage of that unit. Constraint (8) expresses this requirement:

$$\sum_{j=1}^{J} \{\mathrm{SPIN}[j, h, s, y]K[j, h, s] - G[j, h, s, y]\} + \sum_{j \in M} \{R[j, y] - \mathrm{SPIN}[j, h, s, y]\} \times K[j, h, s] \geqslant mD[h, s, y] \tag{8}$$

for all h, s, and y, $M \subseteq \{1, 2, \dots, J\}$ is the set of those plants that have a warm-up lead time of zero.

Storage plants face additional constraints on generation. Their capacity depends on the excess electricity that is generated by other plants and available to be stored. Pumped storage may also face capacity constraints for their storage ponds. Such constraints shall not be dealt with here by assuming that such capacity may be constructed at a very low marginal cost. Equation (9) is the mathematical expression for the pumped storage requirements:

$$\sum_{h=1}^{H(s)} \sum_{j' \in J'} G[j', h, s, y]/q_{j'} \leqslant \sum_{h=1}^{H(s)} \left\{ \left[\sum_{j \notin J'} G[j, h, s, y] \right] - D[h, s, y] \right\} \tag{9}$$

for all s and y. J' is the set of storage/retrieval plants and $0 < q_j < 1$ is the proportion of energy input that can be retrieved using storage plant type j'.

3.5. The objective function

The objective of the planner is to minimize the revenue required to finance the amortization of plants, and all operation, maintenance, and generating cost by choice of the decision variables I, R, ON, SPIN, and G, subject to the constraints given in (1) through (9) over the planning horizon, Y.

The first term shows the discounted value of the revenue required to pay for new capacity. The amount CI[j, y] is required in each year to pay for procuring and scrapping a plant over its expected useful economic life UL(j). Amortization starts when the equipment becomes available for the first time, LI(j) years after the decision to invest into a new plant of type j was made. Capacity whose useful economic life is not finished with the last year, Y, will not be fully financed over the planning horizon. Only that fraction of amortization that corresponds to the fraction of the useful economic life that falls into the planning period enters into the objective function. Were the construction of new plants always fully amortized over the planning horizon, no investment of large and expensive base load plants would be scheduled late in the plan, when fuel efficiency of a few seasons of utilization cannot cover the financial drain of construction and removal:

$$\begin{aligned} \underset{\{I, R, \mathrm{ON}, \mathrm{SPIN}, G\}}{\text{Minimize}} \quad & \sum_{j=1}^{J} \sum_{y=1}^{Y} \sum_{t=y}^{T} \mathrm{CI}[j, t] I[j, y - \mathrm{LI}(j)] \\ & + \sum_{j=1}^{J} \sum_{y=1}^{Y} \mathrm{CR}[j, y] R[j, y] \\ & + \sum_{j=1}^{J} \sum_{y=1}^{Y} \sum_{s=1}^{S} \sum_{h=1}^{H(s)} \mathrm{NC}(s) \mathrm{CON}[j, h, s, y] \times \mathrm{ON}[j, h, s, y] \\ & + \sum_{j=1}^{J} \sum_{y=1}^{Y} \sum_{s=1}^{S} \sum_{h=1}^{H(s)} \mathrm{NC}(s) \mathrm{CG}[j, h, s, y] \times G[j, h, s, y]. \end{aligned} \tag{10}$$

The second term in the objective function gives the total discounted maintenance cost of the system, and the third term gives the total discounted warm-up and cool-down costs of the system. Finally, the fourth term shows the total discounted cost of generation.

The spinning cost is also not explicitly included in the objective function. It should be clear, nonetheless, that the spinning activity is not free. A plant that is kept spinning must generate electricity at a minimum rate of KL[j, h, s] (see constraint (7)). If this electricity is in excess of demand, then the cost to generate at this rate constitutes the spinning cost. Therefore, a plant will not be left in its spinning state when its output is not needed to satisfy demand unless it is either used as a back-up plant or idled in order to avoid high cool-down and warm-up costs.

4. Computational results

The conjecture that the actual shape of the load curve has a significant influence on the optimal choice of the plant mix has lead to the development of the the model presented above. To test this conjecture's potential empirical relevance, the model was run under three different assumptions. In Case 1, demand is represented by the single peaked load–distribution curve of Figure 1. In Case 2, two peaks are present. In Case 3, the load–duration curve that represents both load-distribution curves for Cases 1 and 2 is used. Thus, all three cases face demands having identical total energy requirements and durations of output levels. Although the demand data have been created soleley for this comparison, the load–distribution curves for the two cases have shapes that are representative for those encountered by electric utilities in the real world. In Case 3 it is assumed that getting a plant ready to generate is instantaneous and free of cost. This assumption is implied by all models using traditional load–duration curves.

The solutions were obtained using APEX-III (Control Data Corporation 1979) on a CYBER 175 computer. Various versions of the model were run. While APEX-III provides one of the most efficient solution algorithms available commercially, it should be noted that large versions of the model created some problems. Convergence was often very slow, and small changes in the model formulation resulted in significant changes in computational efficiency. These features are common to all integer and mixed-integer linear programming models and they limit the usefulness of the model because it is difficult to add embellishments to the problem formulation. This is one of the major reasons why we feel that this model serves best as a complement to existing capacity expansion planning models, not as a replacement.

The results of the computations confirm the hypothesis that the shape of the load–distribution curve matters in solving the capacity expansion problem. The results that were obtained for the three cases are stated in Table 1. As the results demonstrate, atmospheric fluid bed combustion plants ($j = 3$) and nuclear light water reactors ($j = 9$) have characteristics that make them desirable in all three cases for meeting base load demand. In none of the three cases does their combined usage rate fall below 55 percent of their combined capacity. Given the long warm-up lead time of nuclear power plants, turning them on and off on a daily basis is not technically feasible in Cases 1 and 2. In Case 3, with warm-up lead times and warm-up and cool-down costs set to zero, nuclear plants could be started up for generating as demand increases. The high investment costs of these plants, however, make it uneconomical to have them sit idle in any scenario.

The atmospheric fluid bed combustion plants are also chosen in all three cases. There are significant differences in how they are dispatched, however. In Case 1, they all kept all day as spinning reserves. In Case 2, one of the two plants is used to meet peak demand only. Consequently, the usage rates relative to combined capacity differ much more widely in the second case. The usage of this plant type in Case 3 is similar to that in Case 2.

Two liquid fuel steam power plants are brought

Table 1
Investment strategies in Cases 1, 2, and 3 (variables $I[j]$)

Plant type (j)	Number of plants of type j		
	Case 1	Case 2	Case 3
1 Conventional coal with wet limestone desulfurication	0	0	0
2 Conventional coal with wet limestone flue gas desulfurication	0	0	0
3 Atmospheric fluid bed combustion	5	2	8
4 Gasification combined cycle	0	0	0
5 Solid solvent refined coal steam	0	2	0
6 Liquid fuel steam (distillate)	0	0	2
7 Liquid fuel steam (residual)	0	0	0
8 Liquid fuel combustion turbine	0	0	0
9 Nuclear light water reactor	11	11	10
10 Liquid fuel combined cycle	5	4	0
11 Underground pumped hydro storage	0	2	0

Table 2
Solution of dispatching problem in Cases 1 and 2

Case 1: Double peaked load–distribution curve

Period	Type $j = 3$		Type $j = 9$		Type $j = 10$	
h	SPIN (plants)	G (MWH)	SPIN (plants)	G (MWH)	SPIN (plants)	G (MWH)
1	5	625	11	7375		
2	5	625	11	5500		
3	5	625	11	6375		
4	5	625	11	8375		
5	5	1400	11	9570	1	30
6	5	1840	11	9570	3	90
7	5	2250	11	9570	5	180
8	5	1280	11	9570	5	150
9	5	2250	11	9570	5	180
10	5	1400	11	9570	1	30
11	5	625	11	9375		
12	5	625	11	9250		

Case 2: Single peaked load–distribution curve

Period	Type $j = 3$		Type $j = 9$		Type $j = 10$		Type $j = 5$		Type $j = 11$	
h	SPIN (plants)	G (MWH)	SPIN (plants)	G (MWH)	SPIN (plants)	G (MWH)	SPIN (plants)	G (MWH)	SPIN (plants)	G (MWH)
1	2	250	11	7750						
2	1	125	11	8395						
3	1	125	11	9570						
4	1	125	11	8875						
5	1	450	11	9570			2	647	1	333
6	2	900	11	9570	2	60	2	637	1	333
7	2	900	11	9570	4	120	2	900	1	510
8	2	900	11	9570	4	120	2	900	1	510
9	2	900	11	9570	1	30	2	500		
10	2	900	11	9570			2	530		
11	2	430	11	9570						
12	2	250	11	8750						

Explanation: h: period of the day, G: level of generation (in MWH), SPIN: number of plants in spinning state.

into the system in Case 3. This plant type has the lowest investment cost per MW of capacity of all eleven plant types and is also cheap to maintain (low CR). On the other hand, it has the second highest generation cost. It is clear from these characteristics that this plant type is not suitable for base load generation but might be economical to provide intermediate and peak load power and serve as spinning reserve in case of an unscheduled outage. In Case 3, the reserve function justified their inclusion in the plant mix. The total power generated by the two plants (megawatt hours, MWH) never exceeded one-third of their combined capacity and was far below that most of the time.

Advanced liquid fuel combined cycle plants are purchased in Cases 1 and 2 in lieu of the liquid fuel steam power plants acquired in Case 3. As Table A.1 shows, their investment cost per MW is relatively low and they can be brought on line to generate quite quickly; their warm-up lead time was assumed to be only four hours. These characteristics make this plant type very suitable for peak load generation; they are used in both cases for this purpose only. They are not used in Case 3 because of their high acquisition and maintenance costs. The short warm-up lead time and low warm-up and cool-down costs are of no advantage in a model that does not consider these operating characteristics. Thus, the dispatching considerations of this model affected the capacity expansion solution.

The load–distribution curves of Cases 1 and 2 bring about not only differences in optimal plant mix relative to Case 3, but between themselves as well. The optimal plant mix in Case 2 included two each of solid solvent refined coal steam plants and underground hydro storage plants. The former are a bit more expensive to acquire and run than a fluid bed combustion plant, but they are cheaper to maintain in the system. Because of the higher generating costs, these plants are not used for base load generation. Instead, both of these plants help meet intermediate and peak load demand.

The underground hydroelectric power stations are relatively expensive to acquire. In addition, their implicit generating cost is high since only two-thirds of the energy used to pump water into storage can be recovered when electricity is generated. On the other hand, if such energy is generated by a plant having low generating costs, nuclear light reactors in this case, the value of these energy losses is minimized. These plants can be started up at a moment's notice to meet the needs of peak loads and forced outages. Of the two storage plants in the system in Case 2, one is dispatched to generate electricity during the peak period, and the other is a reserve in case of an unscheduled outage. This latter use requires only one day to store up. Early morning output from nuclear plants is reduced on subsequent days until this reserve is discharged.

The comparison of the solution to the dispatching problems of Cases 1 and 2 is summarized in Table 2. Case 3 is excluded because its dispatching problem is not explicitly represented by ON and SPIN variables. Table 2 shows that atmospheric fluid bed combustion plants (type $j = 3$) contribute to base demand in both Case 1 and Case 2. In Case 1, however, none of the five plants included in this system is ever turned off. The amount of electricity generated by these plants never drops below 625 MWH. In Case 2, only one of the two plants in the system is never turned off. This plant is not in its spinning state during periods 2 through 5. The role of this plant type in contributing to intermediate and peak demand is, therefore, more pronounced in Case 2 than it is in Case 1. In both cases, liquid fuel combined cycle plants (type $j = 10$) are used for intermediate the peak load demand. In Case 2, solid solvent refined coal steam plants (type $j = 5$) also serve this purpose. Also in Case 2, hydro storage plants ($j = 11$) meet peak load demand and hold nuclear generated power for reserves. Only one of the two storage plants is actually used for generation (during periods 5 through 8). The second plant is held only to provide rapidly available reserve capacity.

5. Conclusions

The computational results clearly demonstrate the potential significance of the shape of the load profile in determining the most economical investment strategy of an electric utility. Cyclical programming and tayloring of parameters and indexes were used to keep such an investigation computationally managable. These technique and results may also be of interest for planners in other utilities that have to produce on demand and encounter strongly fluctuating demand schedules (e.g., mass transit).

The model is not intended to replace successful and established capacity expansion planning models. Like other models, this model gains realism and accuracy in its representation of demand at the expense of realism and accuracy in other areas to remain computationally tractable. We see the purpose of this model as more analogous to the idea of sensitivity analysis. Indeed, this paper tests the potential sensitivity of the results of the popular class of linear capacity expansion models to the assumptions incorporated in the load–duration curve. The answer is that the results may be quite different if these assumptions are relaxed. Even if the model presented above is not used in practice, knowledge of the potential sensitivity of the optimal capacity expansion strategy to changes in the assumptions about demand, should prove useful in assessing the results of other models. It is in this spirit that we present our model and the computational results.

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Computational Appendix

The data used for the three runs have been obtained from the *Technical Assessment Guide* (*TAG*) (EPRI, 1982). Unfortunately, *TAG* includes no information on warm-up lead times and the cost of getting a plant ready to generate electricity. Consequently, assumptions had to be made in order to generate optimal plant mixes for the three cases. We have a fairly good idea bout warm-up lead times for most generating technologies. The difficulty is in determining the warm-up and cool-down costs. The problem was resolved by assuming that warm-up requires the same amount of fuel as generating at the lowest technically feasible limit (KL(j)). The sum of the warm-up and cool-down costs was set equal to the fuel cost of generating at the level KL(j) multiplied by the warm-up lead time. For Case 3, of course, the warm-up lead time is set equal to zero.

TAG (EPRI, 1982) does specify the lower bound on generation (KL(j)) for each plant type. Spinning costs were calculated based on the assumption that to remain ready to generate, a plant would have to produce electricity at that level. In Case 3, this assumption is not relevant since, if the electricity is not needed, the plant can be shut down and restarted later to generate at a moment's notice.

The model was run with eleven different plant types ($j = 11$) to choose from. This includes two different types of a conventional coal steam plant, a solid solvent refined coal and two types of liquid fuel steam power plants, a nuclear power plant, and two types of combined cycle plants. Also included among the eleven plant types is an underground pumped hydro storage system. It was argued above that the desirability of storage facilities is quite likely to change significantly as the shape of the load-distribution curve changes. The most important parameters of the eleven plant types are summarized in Table A.1. The annual investment costs have been calculated based on an assumed discount rate of 12 percent and the useful economic life of each plant as given in EPRI (1982).

Although the demand for electricity has no real world source, the load-distribution curves in Figure 1 are representative for the kind of demand variations facing an electric utility. The computation uses only one period. The results represent, therefore, the plant mix that is desirable in the long run under the given demand conditions. The runs ignore short term considerations that arise because of plants that are present when the season starts. The problem that is presented here has 696 constraints and 286 variables. Only 12 of these variables are restricted to integer values. As ex-

Table A.1
Summary of plant statistics (EPRI, 1982) [a]

Plant type [b]	LS (hrs)	K (MW)	KL (MW)	CG ($/MWH)	CR/MW ($/unit-yr)	CON ($/Unit)	CSPIN ($/hr)	CI/MW ($/Unit-yr)
$j=1$	8	450	125	20	17,000	19,500	2,500	137,333
$j=2$	12	450	125	19	17,000	28,200	2,400	138,000
$j=3$	8	450	175	19	9,333	26,900	3,400	113,111
$j=4$	8	890	400	17	20,112	53,800	6,700	143,708
$j=5$	8	450	125	23	3,778	23,200	2,900	116,667
$j=6$	6	450	125	67	1,889	50,600	8,400	76,667
$j=7$	6	450	125	51	1,889	38,200	6,400	89,556
$j=8$	0	73	0	97	411	0	0	295,890
$j=9$	23	870	500	10	10,230	175,100	4,900	164,138
$j=10$	4	280	30	52	5,714	6,300	1,600	99,643
$j=11$	0	654	333	1	1,682	0	70	136,239

[a] Numbers have been rounded. CI has been calculated based on the economic life of each plant and a discount rate of 12 percent. Dismantling costs are not included. CON and CSPIN are based on assumptions (see text). No information is available from TAG (EPRI, 1982).

[b] For explanation on plant types, see Table 1.

plained in the text, however, at the optimum, all variables will assume integer values.

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